May 19, 2022
Artificial Intelligence in Research and Applications Seminar (AIRA)
of the Group for Engineering of Intelligent Systems and Technologies,
Faculty of Physics, Astronomy and Applied Computer Science JU

Investigating and facilitating human geometric cognition through VR/AR technologies

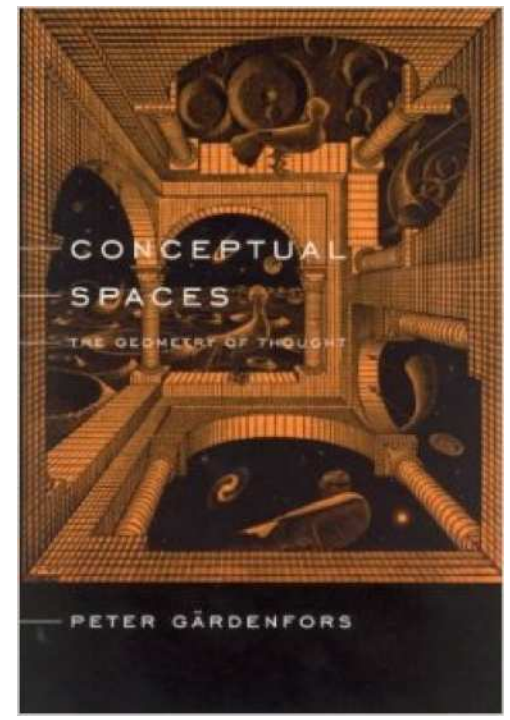
Mateusz Hohol

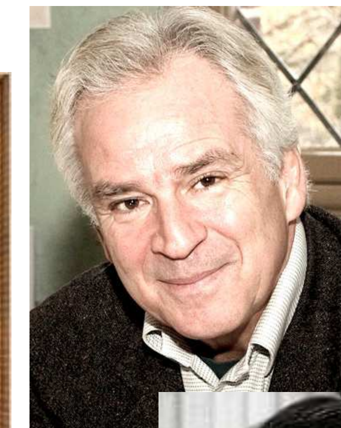
Copernicus Center for Interdisciplinary Studies
Jagiellonian University

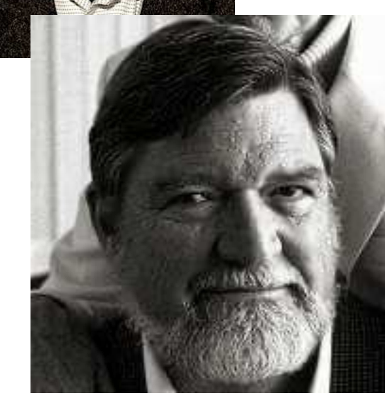
JAGIELLONIAN UNIVERSITY IN KRAKÓW
Copernicus Center

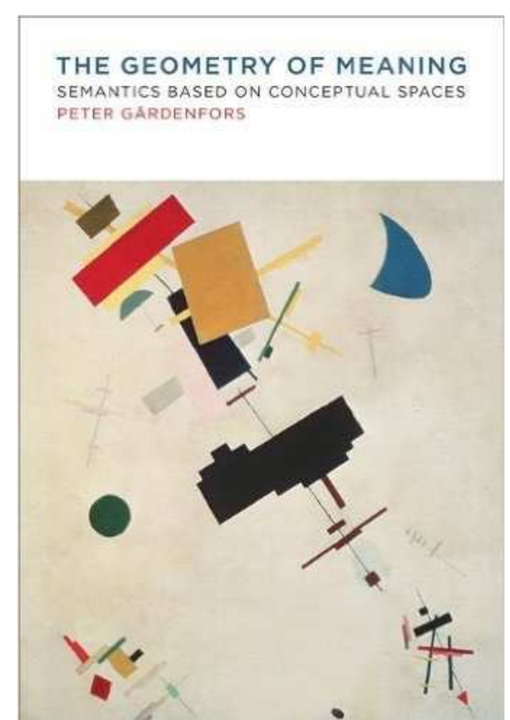
Geometry as explanans

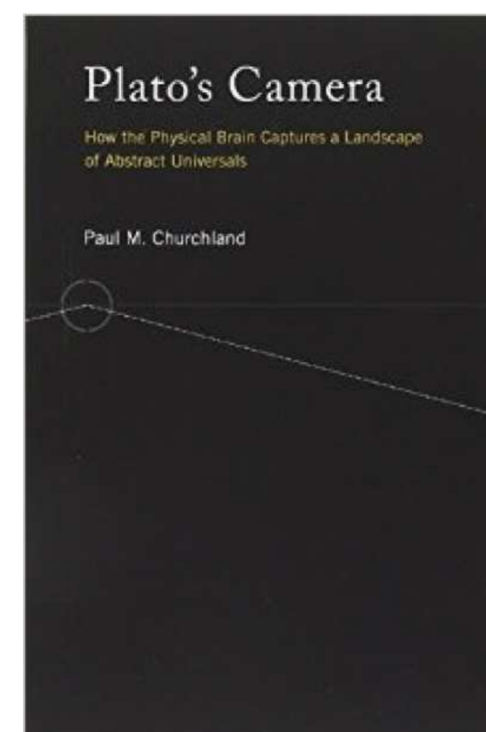
- Usually computational cognitive scientists use various geometric structures for modeling cognitive representations and processes
- Credible accounts:
  - Paul Churchland (2012): conceptual maps
  - Peter Gärdenfors (2004, 2014): conceptual spaces





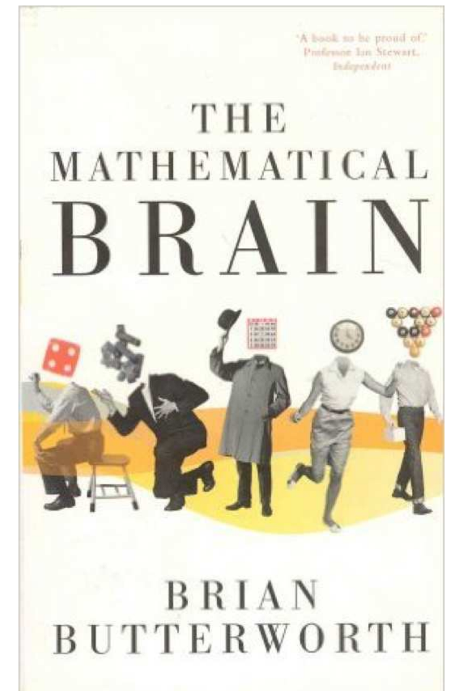


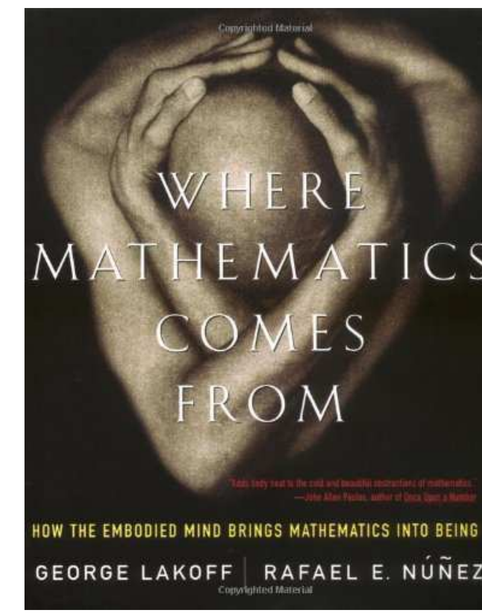


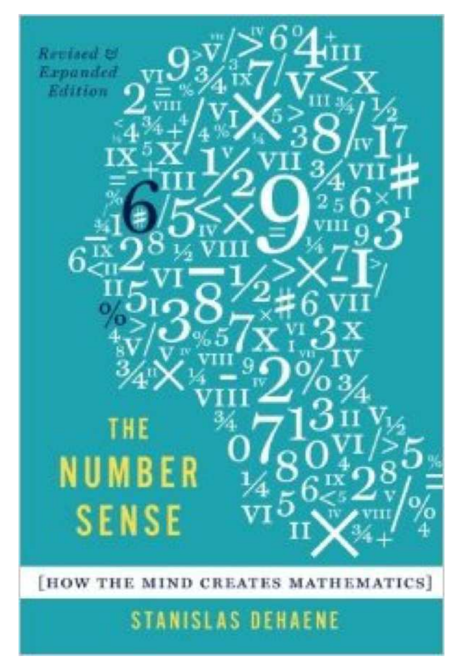


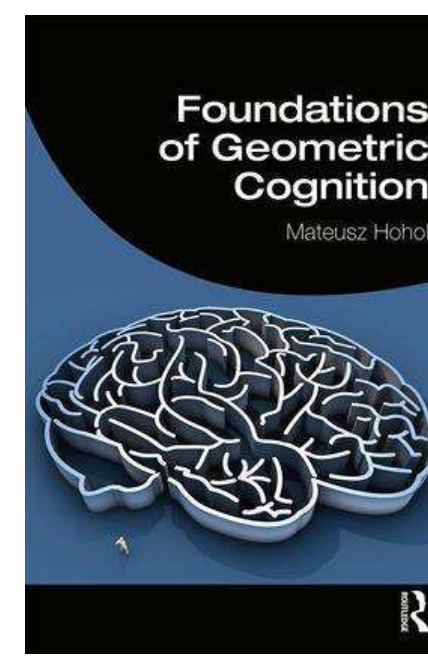
Geometry as explanandum

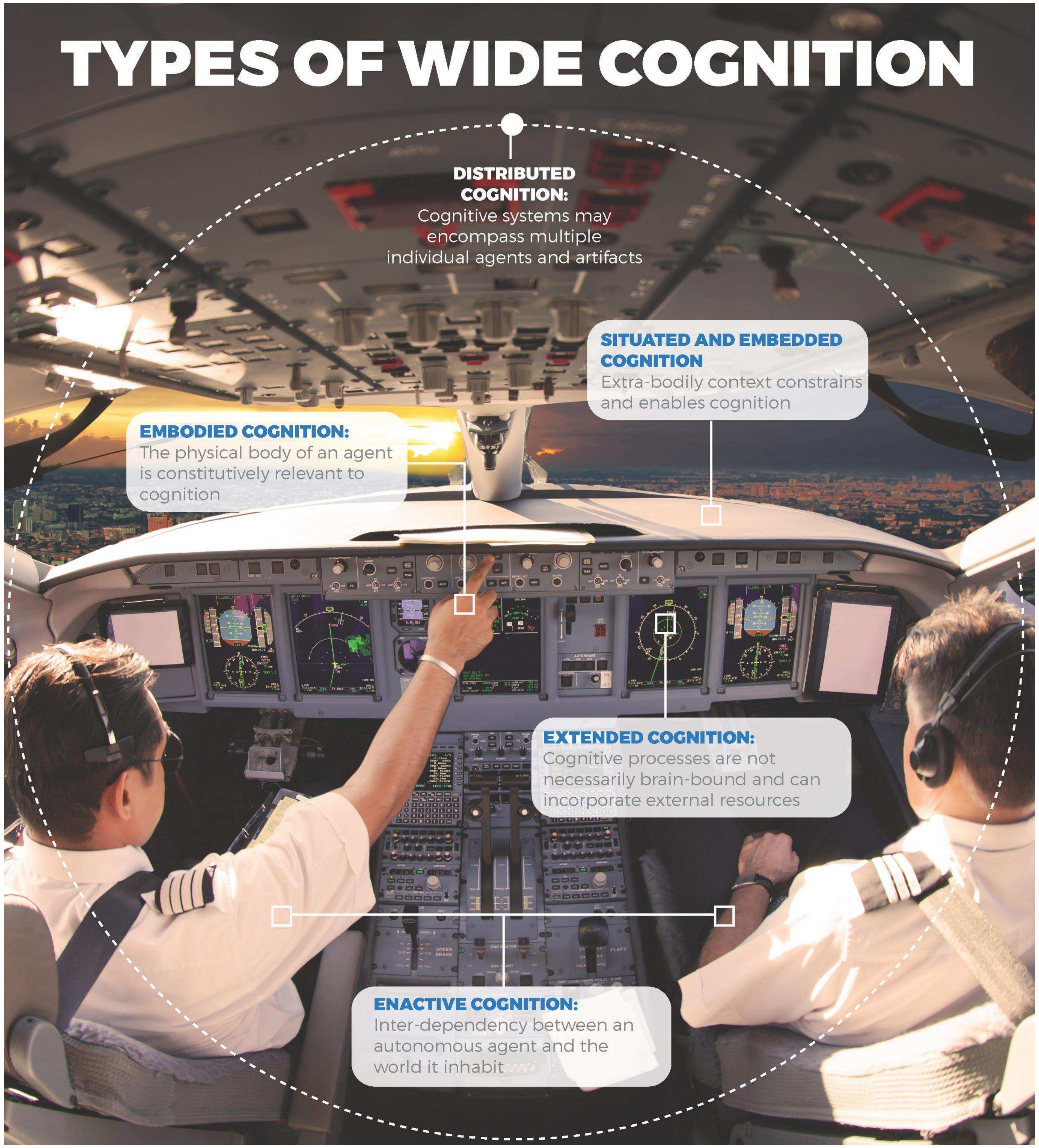
- Understanding cognitive foundations of geometry is essential in light of educational reports (e.g., PISA/OECD) showing that Western students are less proficient in geometry in juxtaposition to other fields of mathematical education
- Like any human intellectual enterprise, Euclidean geometry (and whole mathematical activity) emerges from Lowe-level cognitive processes grounded in the activity of our brains and bodies, as well as our interactions with natural and artificial component parts of the environment











Cognition is framed as

- embodied
- enactive
- embedded/situated
- extended
- distributed

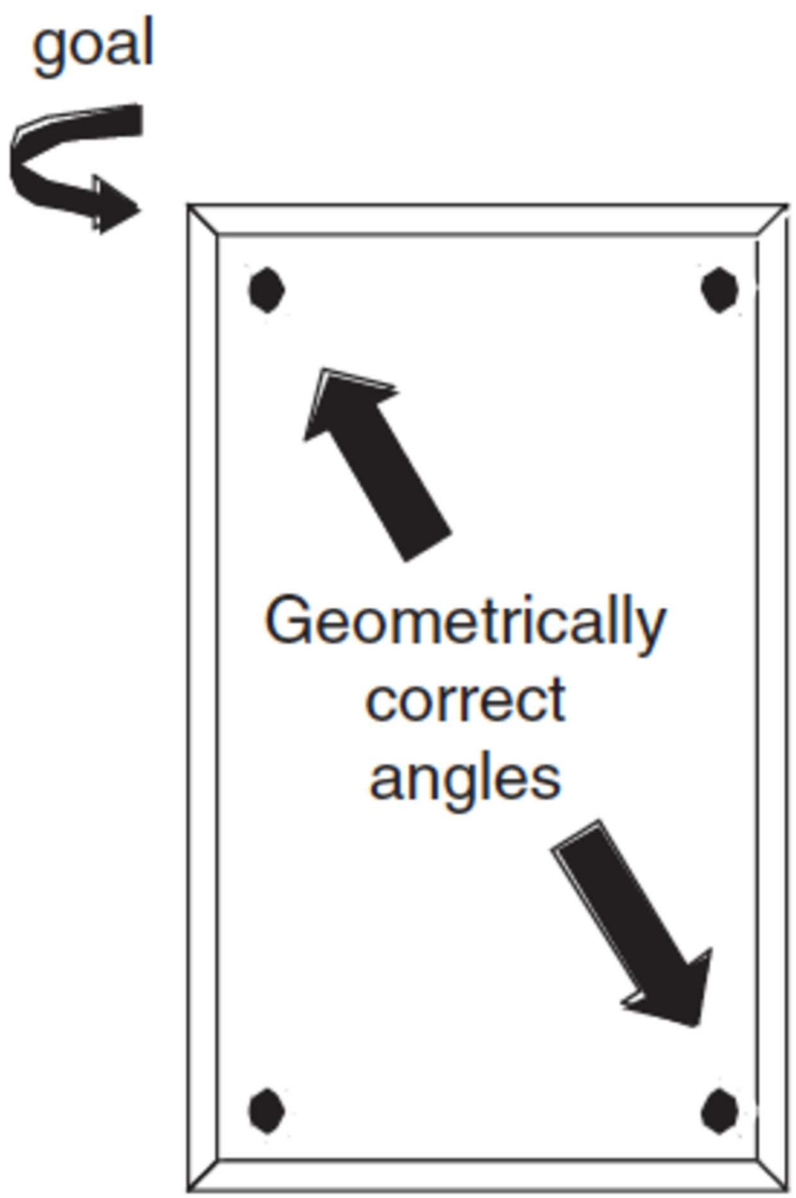
From Wide Cognition to Mechanisms: A Silent Revolution

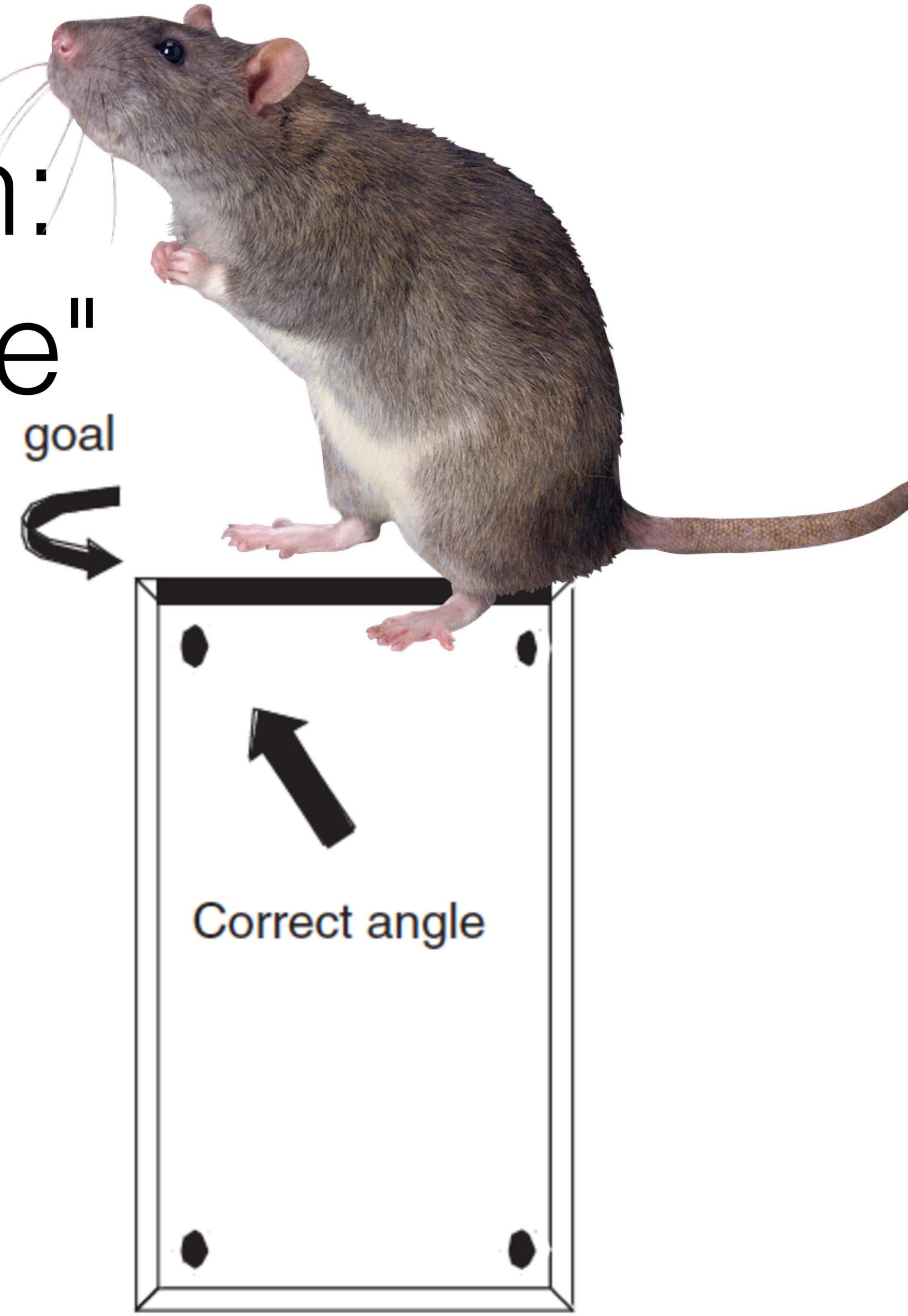
*Marcin Miłkowski[1], Robert Clowes[2], Zuzanna Rucińska[1], Aleksandra Przegalińska[3], Tadeusz Zawidzki[4], Joel Krueger[5], Adam Gies[6], Marek McGann[7], Łukasz Afeltowicz[8], Witold Wachowski[9], Fredrik Stjernberg[10], Victor Loughlin[11] and Mateusz Hohol[1,12]**

frontiers in Psychology

2018 | Volume 9 |

Spatial navigation:
"Geometric module"

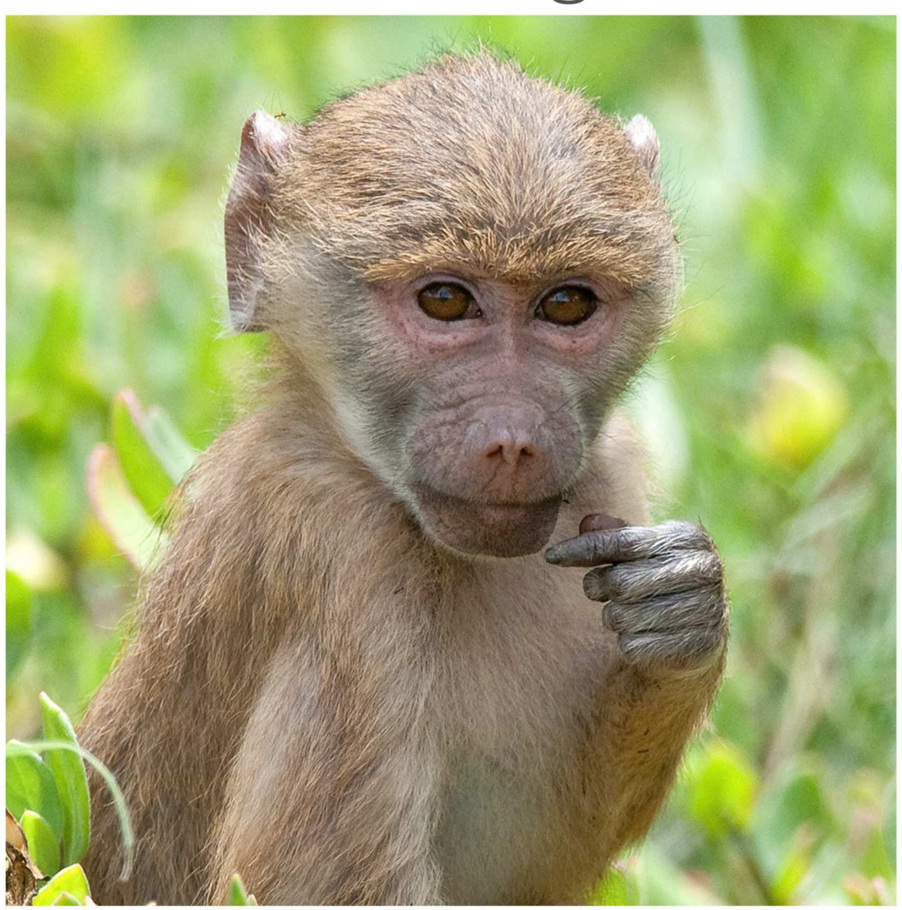




Sensitivity: distance & direction (Cheng, 1980)

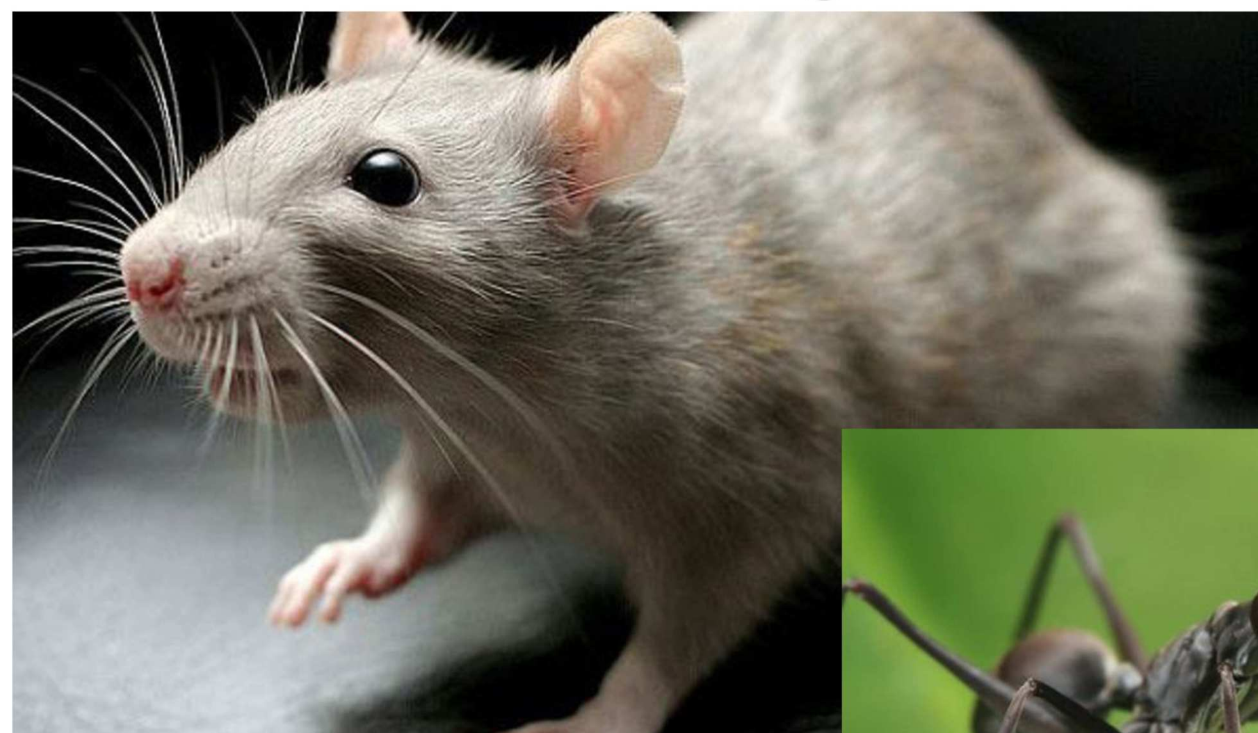
birds: Vallortigara et al, 1990

children: Hermer & Spelke, 1994

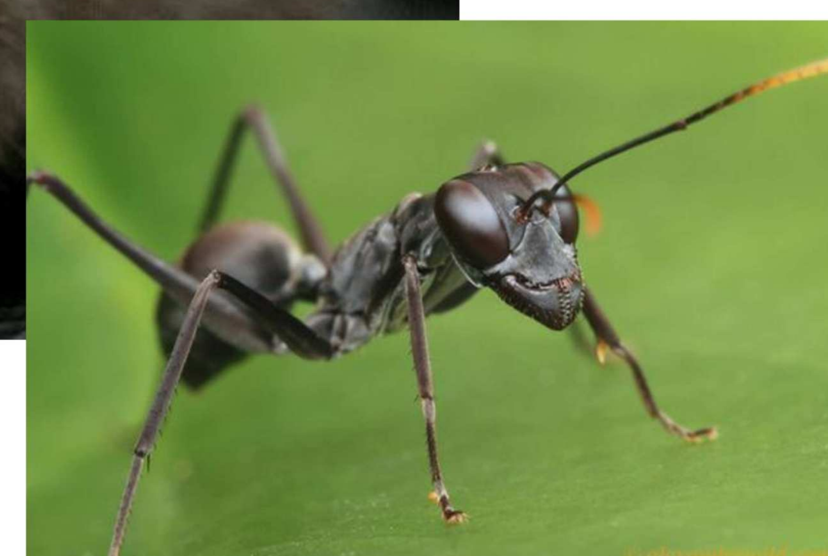


primates: Gouteux, 1999

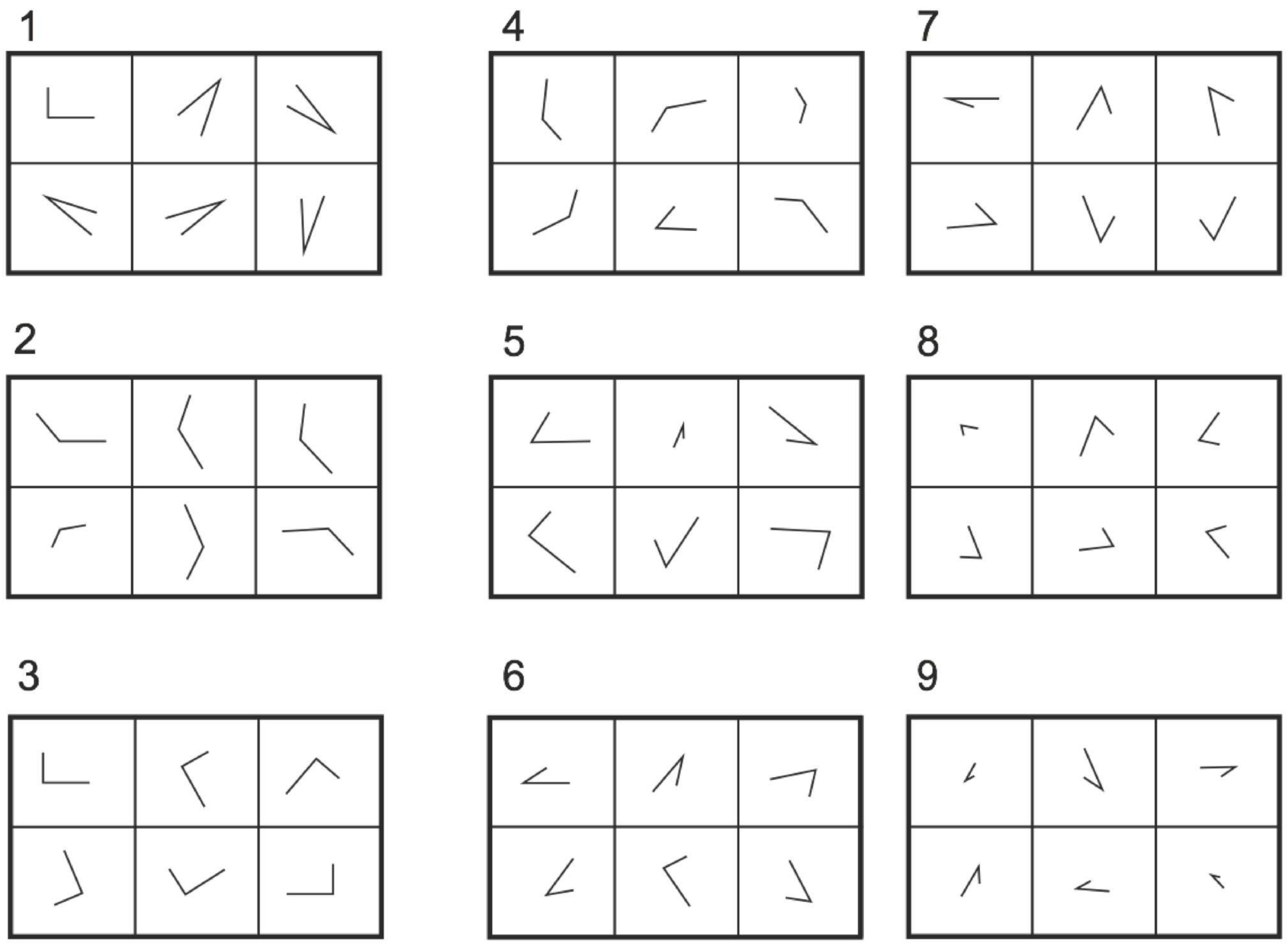
rodents: Cheng, 1986



insects: Wystrach & Beugnon, 2009



Object recognition: Deviant detection paradigm

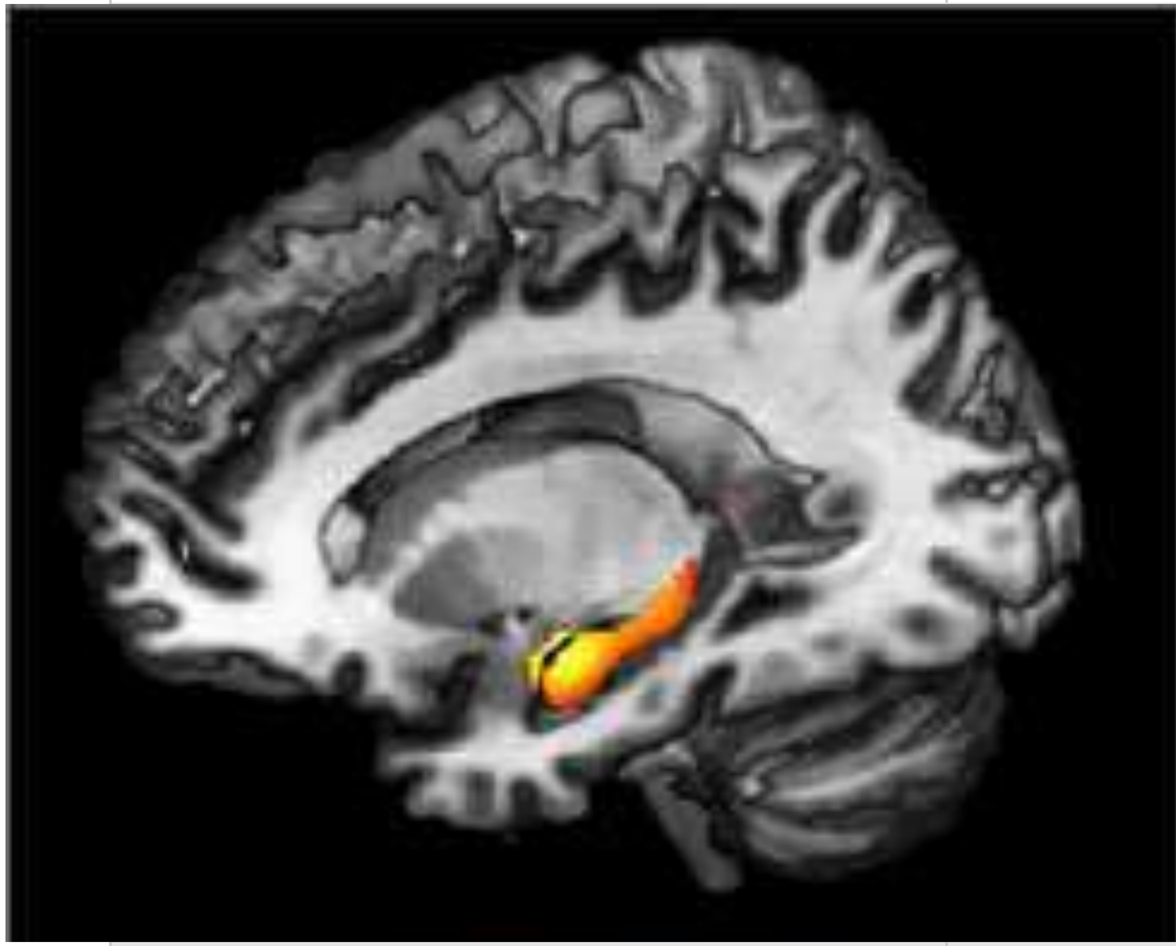


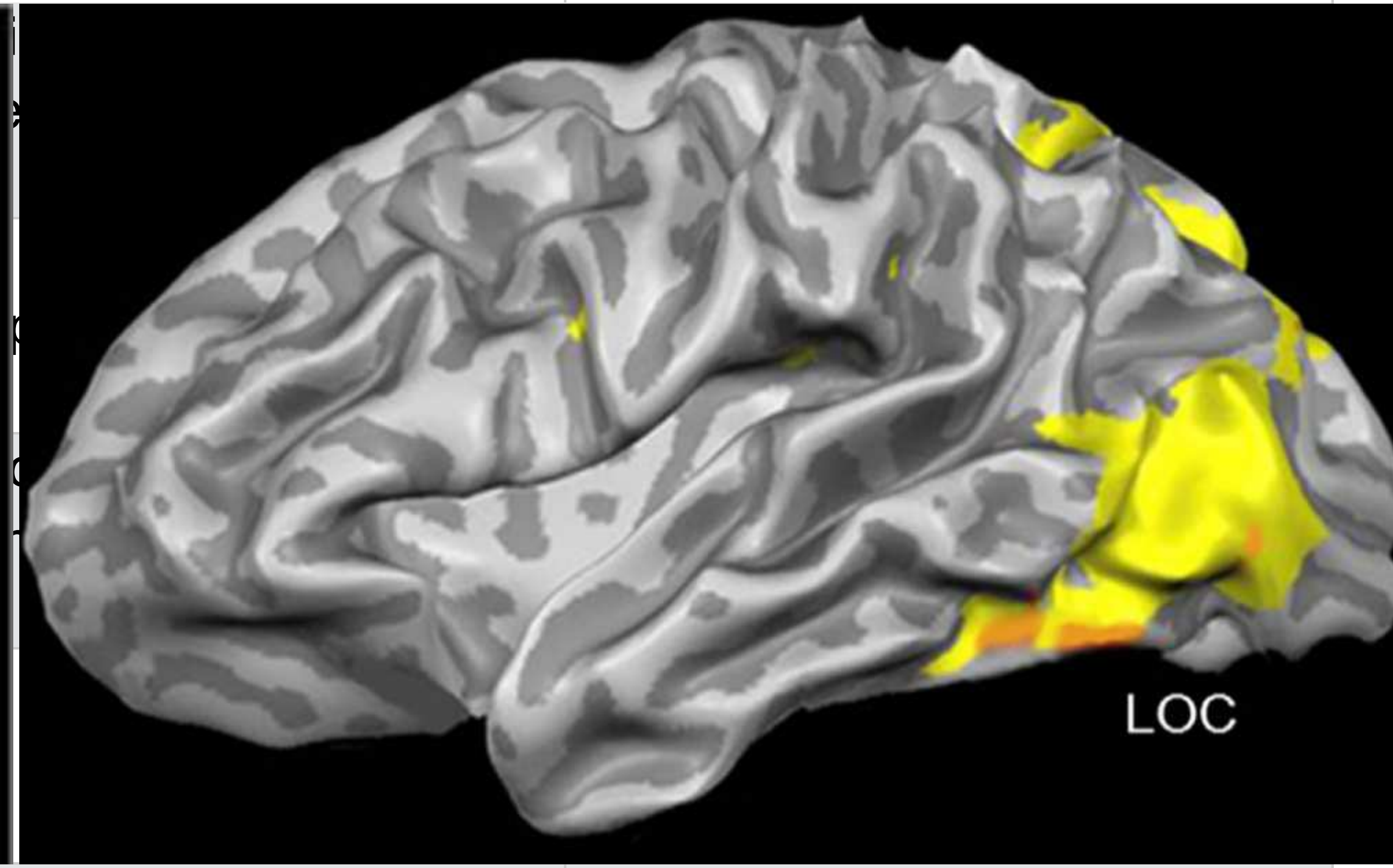
Sensitivity: length & angle

Various tested groups, e.g., Amazonian Indigene people (Dehaene et al., 2006); infants (Izard & Spelke, 2009)

34 (2010) 863–884
COGNITIVE SCIENCE
A Multidisciplinary Journal

Beyond Core Knowledge: Natural Geometry Elizabeth Spelke, Sang Ah Lee, Véronique Izard	**The core system of layout geometry**	**The core system of object geometry**

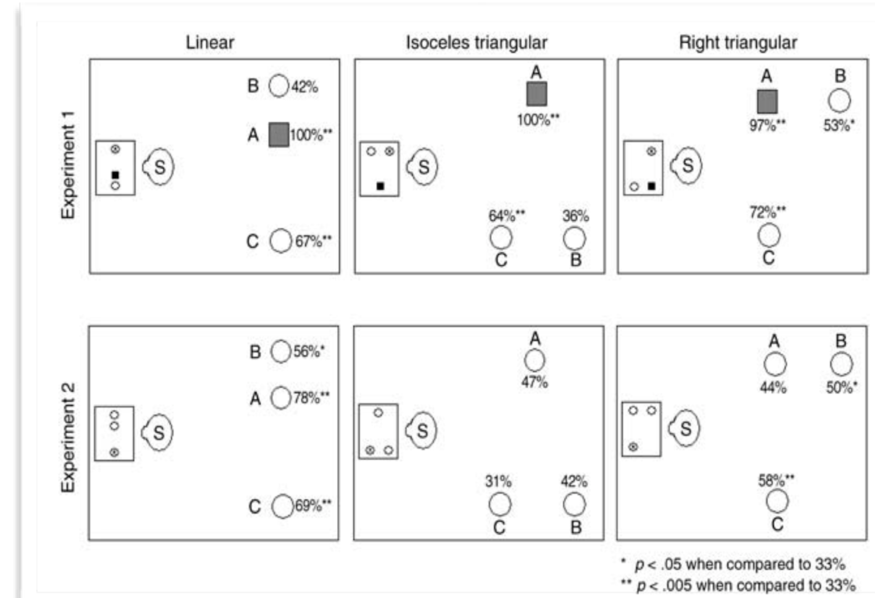




Brain structures involved in typical tasks	hippocampus, entorhinal cortex, occipital place area, retrosplenial cortex	lateral occipital cortex, occipitotemporal sulcus, fusiform gyrus
Association with systems of visual processing	dorsal stream (“where?” system)	ventral stream (“what?” system)
Linguistic expression	spatial prepositions: an object plays the role of “figure” or “ground”	object nouns: an object is named as belonging to a category

Toward more general representational system of geometry

- The developmental shift resulting in toddler's performance becoming similar to that of adults occurs between 5 and 7 years (at that age, children become able to flexibly combine layout and object geometry
- Connection between children's capacity to generate spatial phrases, especially involving concepts of LEFT and RIGHT (Hermer-Vazquez et al., 2001).
- Thanks to the familiarization with maps, scale objects, etc. children start to represent navigable spatial layouts structured cognitively by relationships of distance (proximal-distal) and sense (left-right), as arrangements also characterized virtually by angular properties (Spelke, Lee & Izard, 2010).
- Growing experience with rotating (both physical and mental) of small objects and figures may facilitate understanding them not only as forms characterized solely by angle and length relationships, but also as layouts that can be explored from different perspectives (sense/direction property)



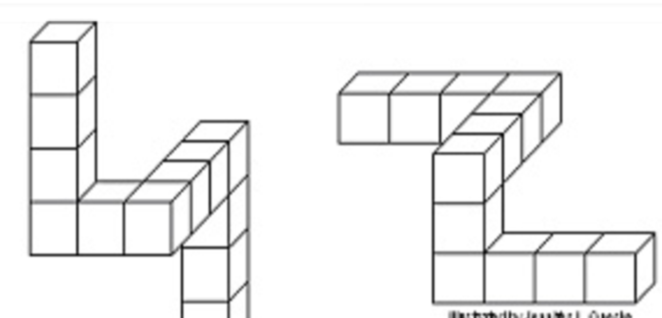
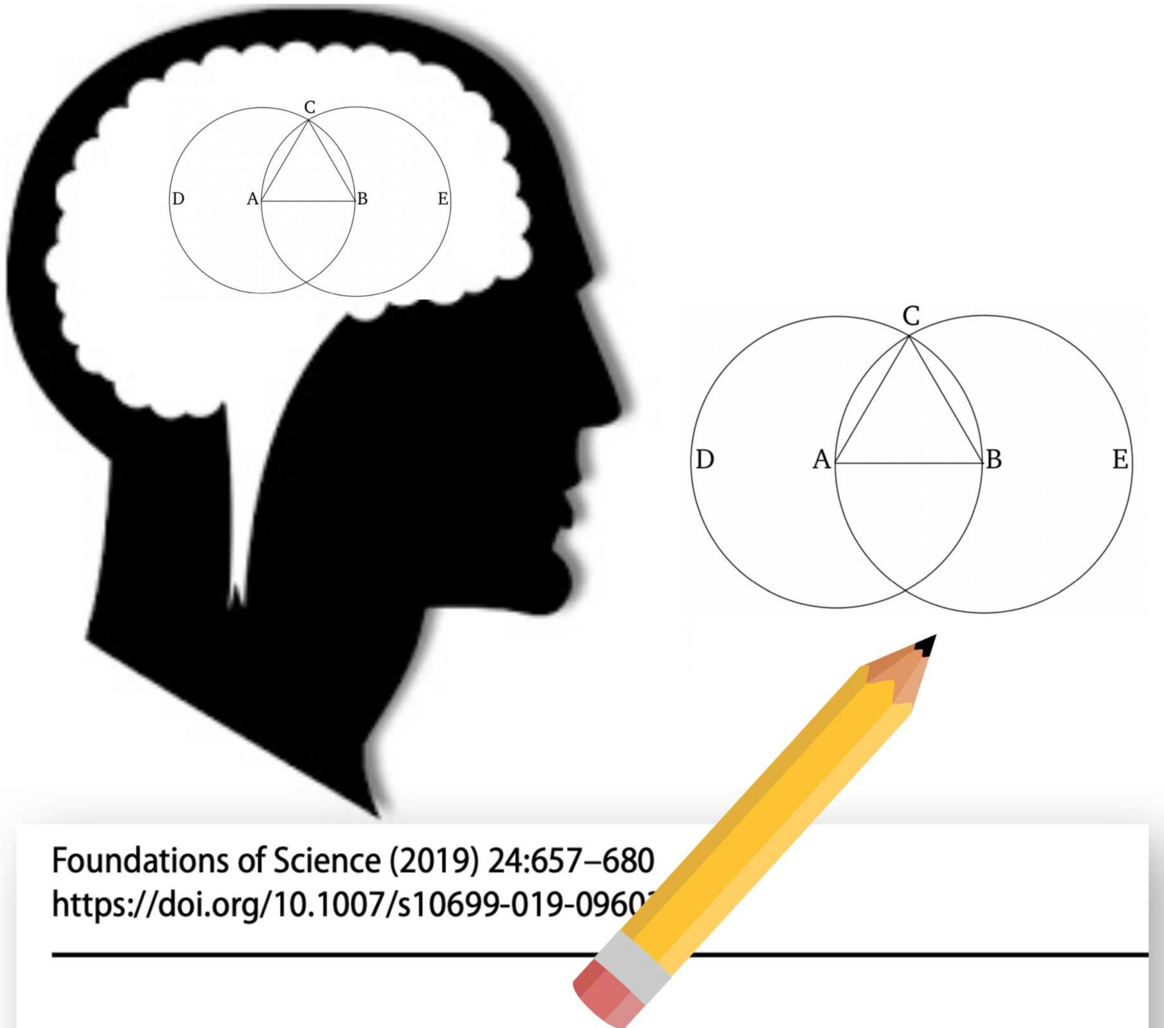


Figure 1: Based on Shepard & Metzlar's 'Mental Rotation Task'

Cognitive artifacts

- They "maintain, display, or operate upon information in order to serve a representational function and that affect human cognitive performance" (Norman, 1991).
- They are building-blocks of "a cognitive niche" which allows cognitive agents to interact, collaborate, and solve problems collectively.
- Cognitive artifacts are building blocks of cognitive niches; ("an animal-built physical structure that (…) enhance problem-solving, and (…) make possible whole new forms of thought and reason," Clark, 2006))



Foundations of Science (2019) 24:657–680
https://doi.org/10.1007/s10699-019-0960

Cognitive Artifacts for Geometric Reasoning

Mateusz Hohol[1,2] · Marcin Miłkowski[2]

frontiers in Integrative Neuroscience

PERSPECTIVE
published: 27 October 2021
doi: 10.3389/fnint.2021.734930

Making Cognitive Niches Explicit: On the Importance of External Cognitive Representations in Accounting for Cumulative Culture

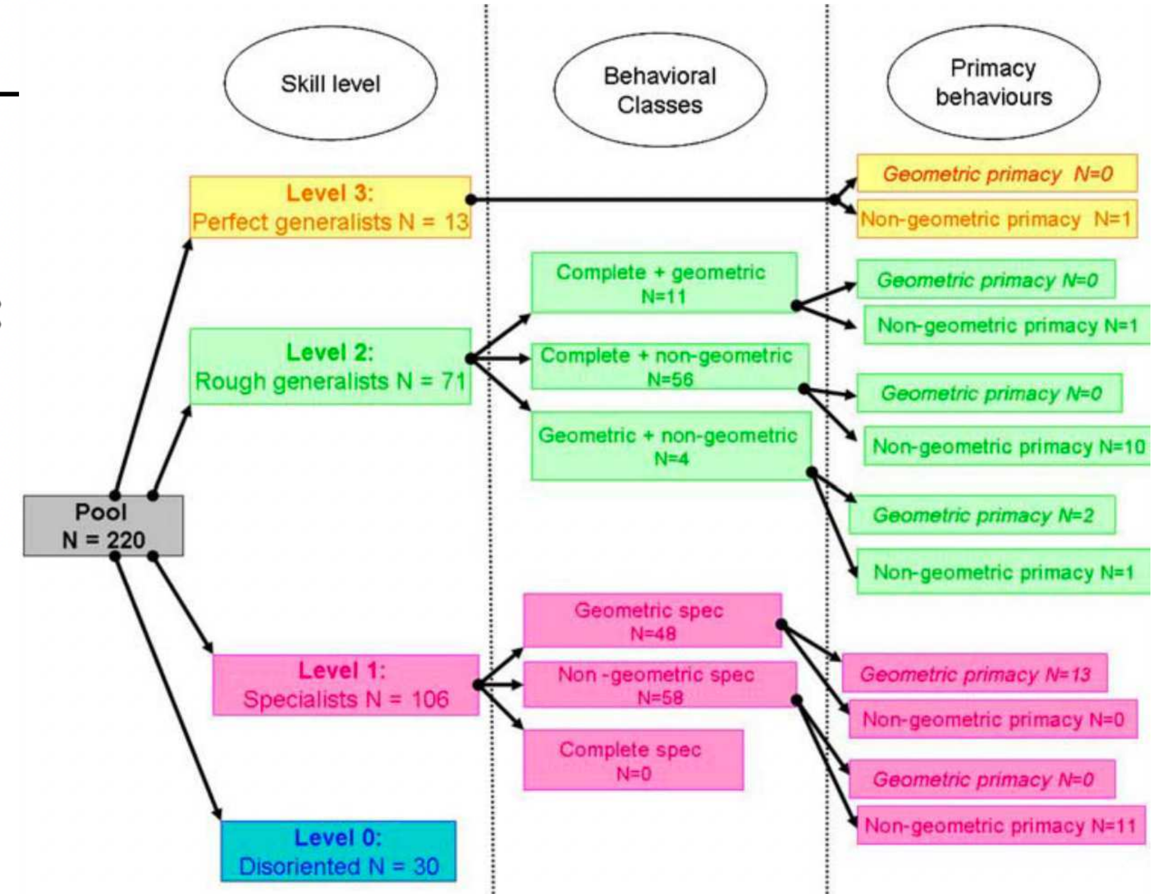
Mateusz Hohol[1†], Kinga Wołoszyn[2†] and Bartosz Brożek[1,3]*

Anim Cogn (2010) 13:157–174
DOI 10.1007/s10071-009-0255-7

ORIGINAL PAPER

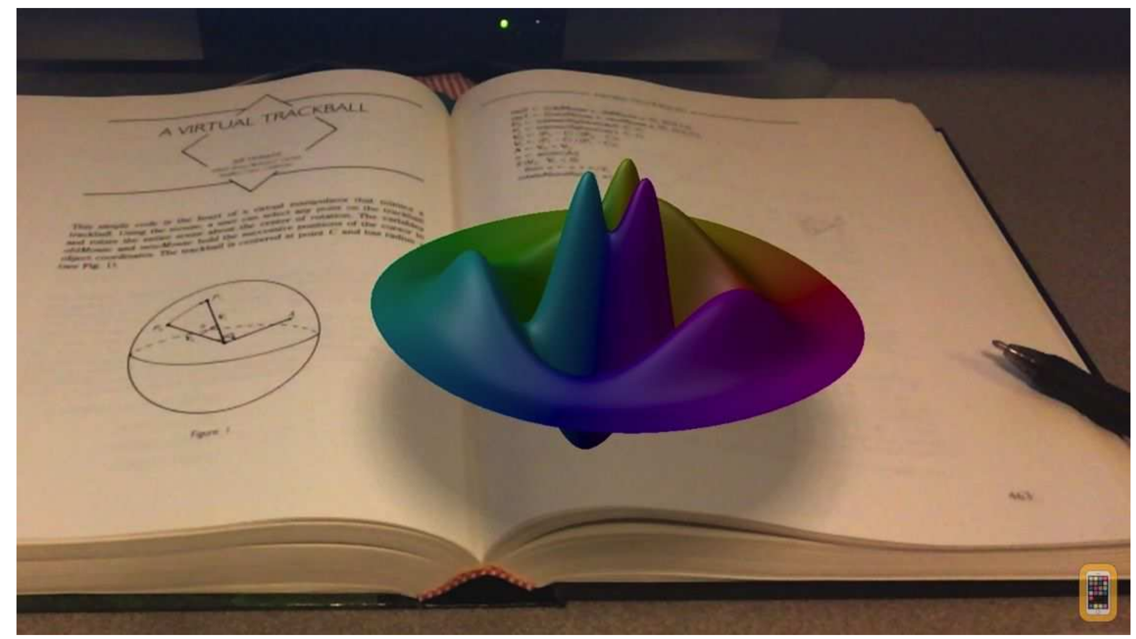
Encoding geometric and non-geometric information: a study with evolved agents

Michela Ponticorvo · Orazio Miglino



- It is possible to evolve agents with different spatial skills by varying the frequency with which they are exposed to different classes of stimuli during their evolution.
- Agents that evolve in environments providing balanced exposure to geometric and non-geometric cues acquire the ability to use both kinds of cue. Agents that are exposed primarily to a single class of cue show primacy.
- Geometric primacy, non-geometric primacy or successful integration between the two classes of information depend on the relative frequencies at which organisms are exposed to these information during their evolution and development

VR and AR: Virtual and Augmented Realities



Numerous VR studies on geometry-based navigation in humans

Functional Similarities in Spatial Representations Between Real and Virtual Environments

BETSY WILLIAMS, GAYATHRI NARASIMHAM, CLAIRE WESTERMAN, JOHN RIESER, and BOBBY BODENHEIMER
Vanderbilt University

Psychonomic Bulletin & Review
2008, 15 (2), 322-327
doi: 10.3758/PBR.15.2.322

Spatial memories of virtual environments: How egocentric experience, intrinsic structure, and extrinsic structure interact

JONATHAN W. KELLY AND TIMOTHY P. MCNAMARA
Vanderbilt University, Nashville, Tennessee

Cognition 109 (2008) 281–286
Contents lists available at ScienceDirect
Cognition
journal homepage: www.elsevier.com/locate/COGNIT
ELSEVIER
COGNITION

Brief article

The shape of human navigation: How environmental geometry is used in maintenance of spatial orientation

Jonathan W. Kelly [a,*], Timothy P. McNamara [a], Bobby Bodenheimer [b], Thomas H. Carr [c], John J. Rieser [d]

Cognitive Maps: Some People Make Them, Some People Struggle

Steven M. Weisberg[1] and Nora S. Newcombe[2]
[1]Center for Cognitive Neuroscience, University of Pennsylvania, and
[2]Department of Psychology, Temple University

Journal of Experimental Psychology:
Learning, Memory, and Cognition
2010, Vol. 36, No. 5, 1097–1107

0278-7393/10/$12.00 DOI: 10.1037/a0019938

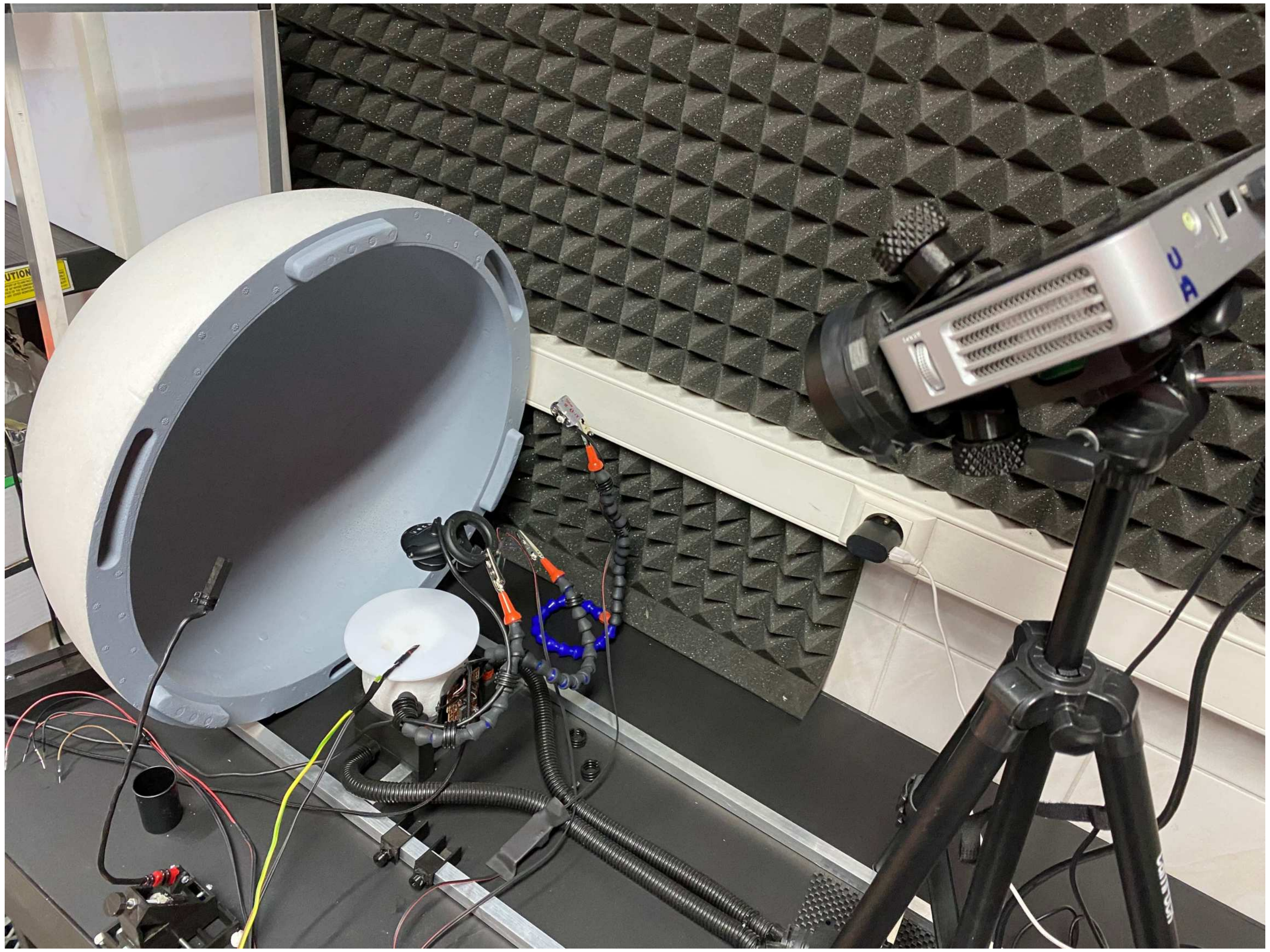
Spinning in the Scanner: Neural Correlates of Virtual Reorientation

Jennifer E. Sutton
Brescia University College

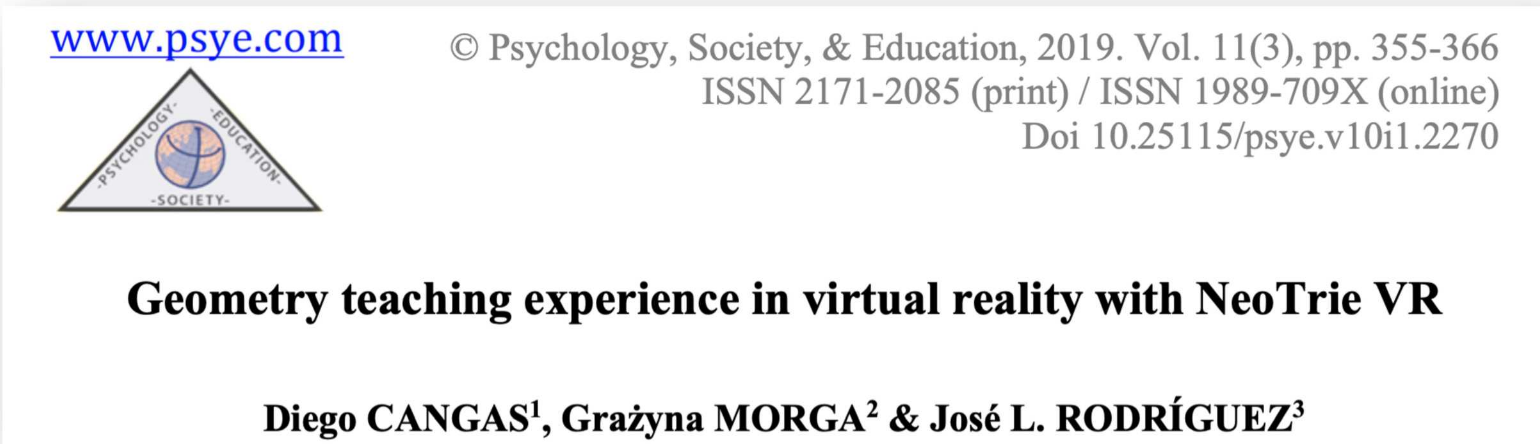
Marc F. Joanisse
The University of Western Ontario

Nora S. Newcombe
Temple University

Ongoing „VR-like” study on insects



NeoTrie VR

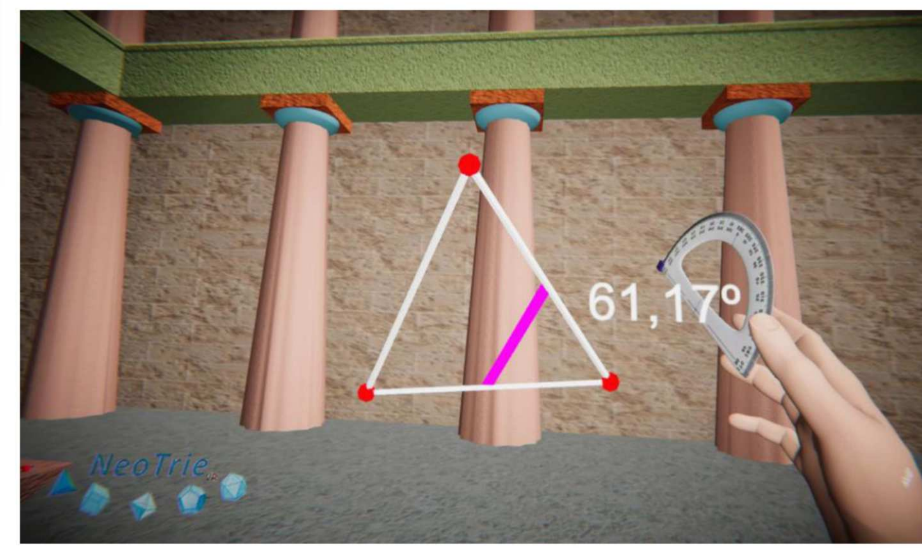


www.psye.com
© Psychology, Society, & Education, 2019. Vol. 11(3), pp. 355-366
ISSN 2171-2085 (print) / ISSN 1989-709X (online)
Doi 10.25115/psye.v10i1.2270

Geometry teaching experience in virtual reality with NeoTrie VR

Diego CANGAS[1], Grażyna MORGA[2] & José L. RODRÍGUEZ[3]

- By Virtual Dor (a spin-off of the University of Almería
- Tests in Primary School in Żernica, Poland
- 11, 12 and 14 years old children
- Promising, but (so far) the lack of rigorous testing of efficiency of interventions



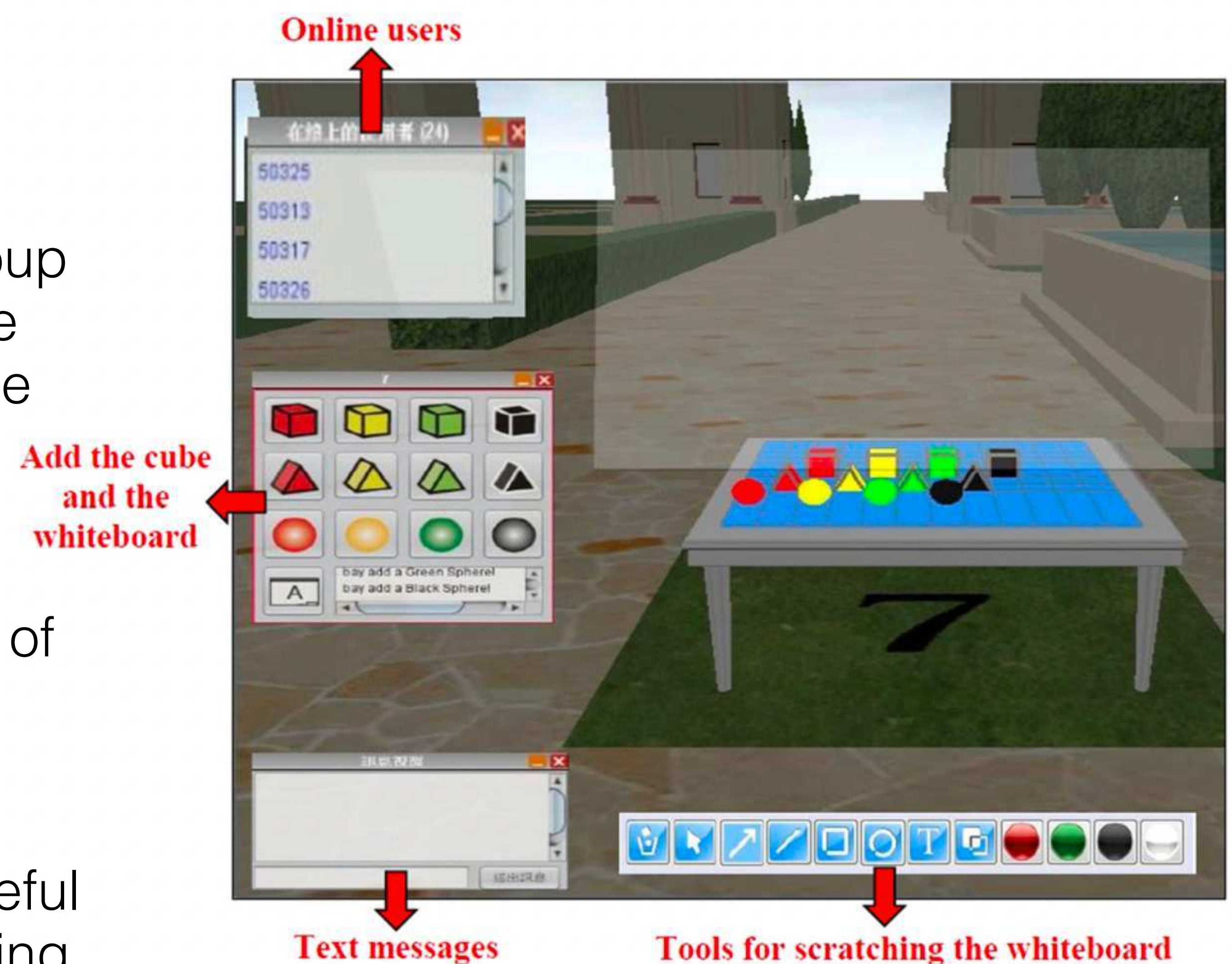
Interactive Future Mathematics Classroom (IFMC)

Computers & Education 62 (2013) 308–319
Contents lists available at SciVerse ScienceDirect
Computers & Education
journal homepage: www.elsevier.com/locate/compedu

Analysis of peer learning behaviors using multiple representations in virtual reality and their impacts on geometry problem solving

Wu-Yuin Hwang [1], Shih-Shin Hu*

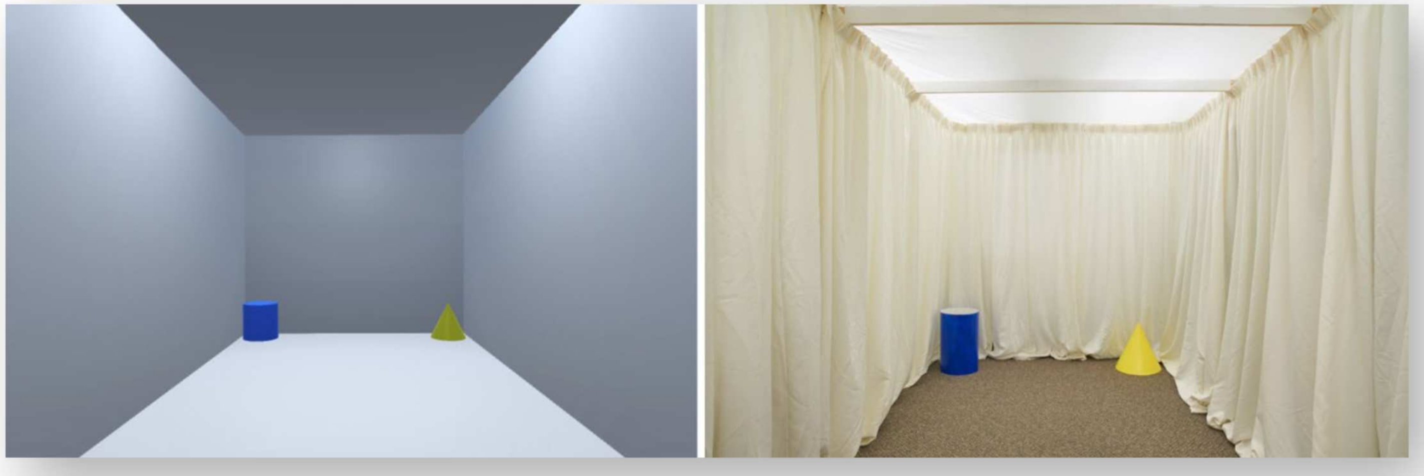
- One eight-week experiment was conducted and the results showed that the experimental group (**>10 year old**) using the IFMC performed significantly better than the control group on geometric learning achievement.
- The learning behaviors of the experimental group in the two kinds of geometric problems, volume and surface area calculation, were different due to the problems' varying difficulty levels.
- Moreover, various learning behaviors with multiple representations lead to different types of strategies for geometric problem solving in the IFMC.
- Learning behaviors in the IFMC were found useful to facilitate geometric problem solving by sharing ideas and exploring multiple representations.



.

Issues of VR

- Kimura et al., 2017: Comparing behavior in Hermer & Spelke’s like task in physical and VR setting
- Following a disorientation procedure, people could reorient by using either the geometry of the room and/or the distinct features in the corners.
- Test trials in which the different spatial cues were manipulated revealed participants encoded features and geometry in both the real and VR rooms.
- However, participants in the VR room showed less facility with using geometry.
- Thus: VR is not so ecologically valid as previously thought
- Desktop environments as not so poor as considered (Zhao et al., 2020)
- Also: Ethical restrictions for VR studies in younger children



SCIENTIFIC REPORTS

OPEN

Orientation in Virtual Reality Does Not Fully Measure Up to the Real-World

Received: 6 July 2017
Accepted: 7 December 2017

Kazushige Kimura[1], James F. Reichert[2], Ashley Olson[2], Omid Ranjbar Pouya[1], Xikui Wang[3], Zahra Moussavi[1] & Debbie M. Kelly[1,2]

SPATIAL COGNITION & COMPUTATION
2020, VOL. 20, NO. 4, 328–363
https://doi.org/10.1080/13875868.2020.1817925

Taylor & Francis
Taylor & Francis Group

Check for updates

Desktop versus immersive virtual environments: effects on spatial learning

Jiayan Zhao[a], Tesalee Sensibaugh[b*], Bobby Bodenheimer[c†], Timothy P. McNamara[d†], Alina Nazareth[e†], Nora Newcombe[f†], Meredith Minear[b#], and Alexander Klippel[a]

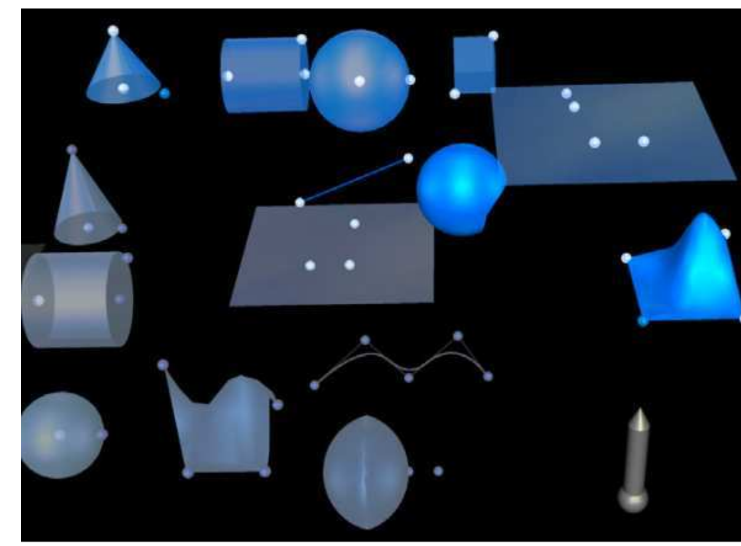
AR: Construct3D

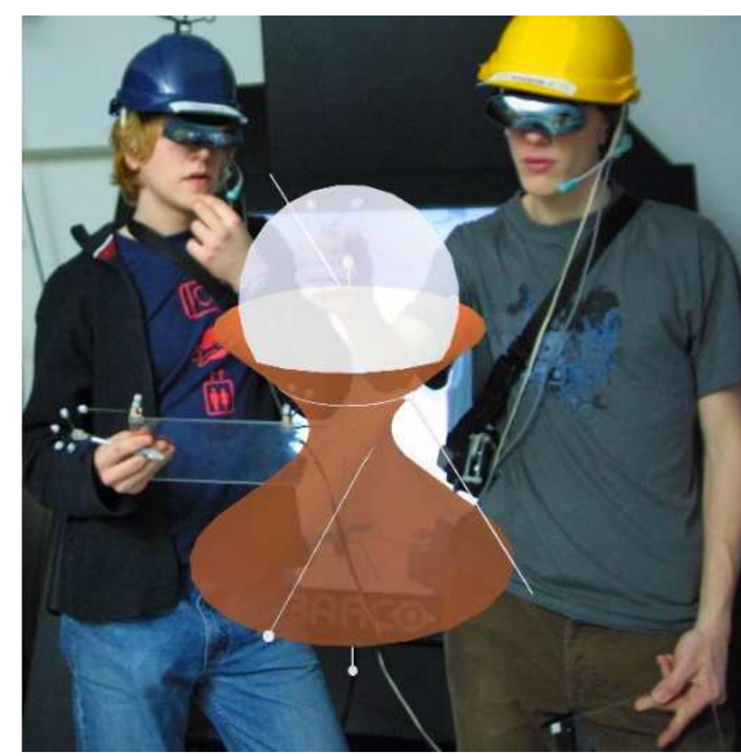
Designing Immersive Virtual Reality for Geometry Education

Hannes Kaufmann[1]
Institute of Software Technology and Interactive Systems
Vienna University of Technology

Dieter Schmalstieg[2]
Institute for Computer Graphics and Vision
Graz University of Technology

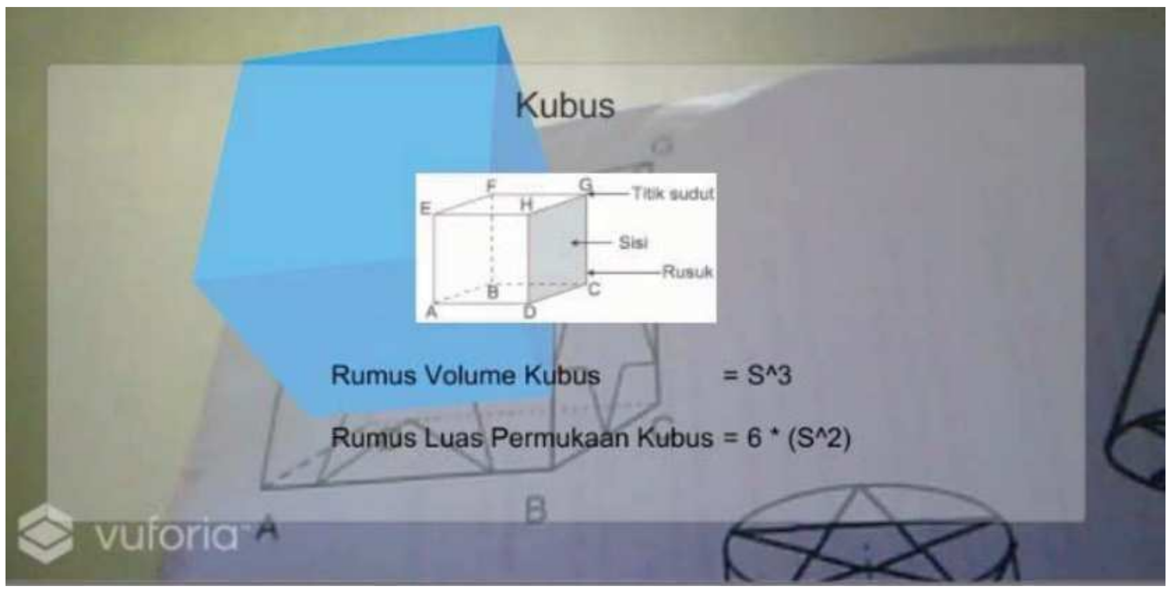
Proceedings of the IEEE Virtual Reality Conference (VR'06)

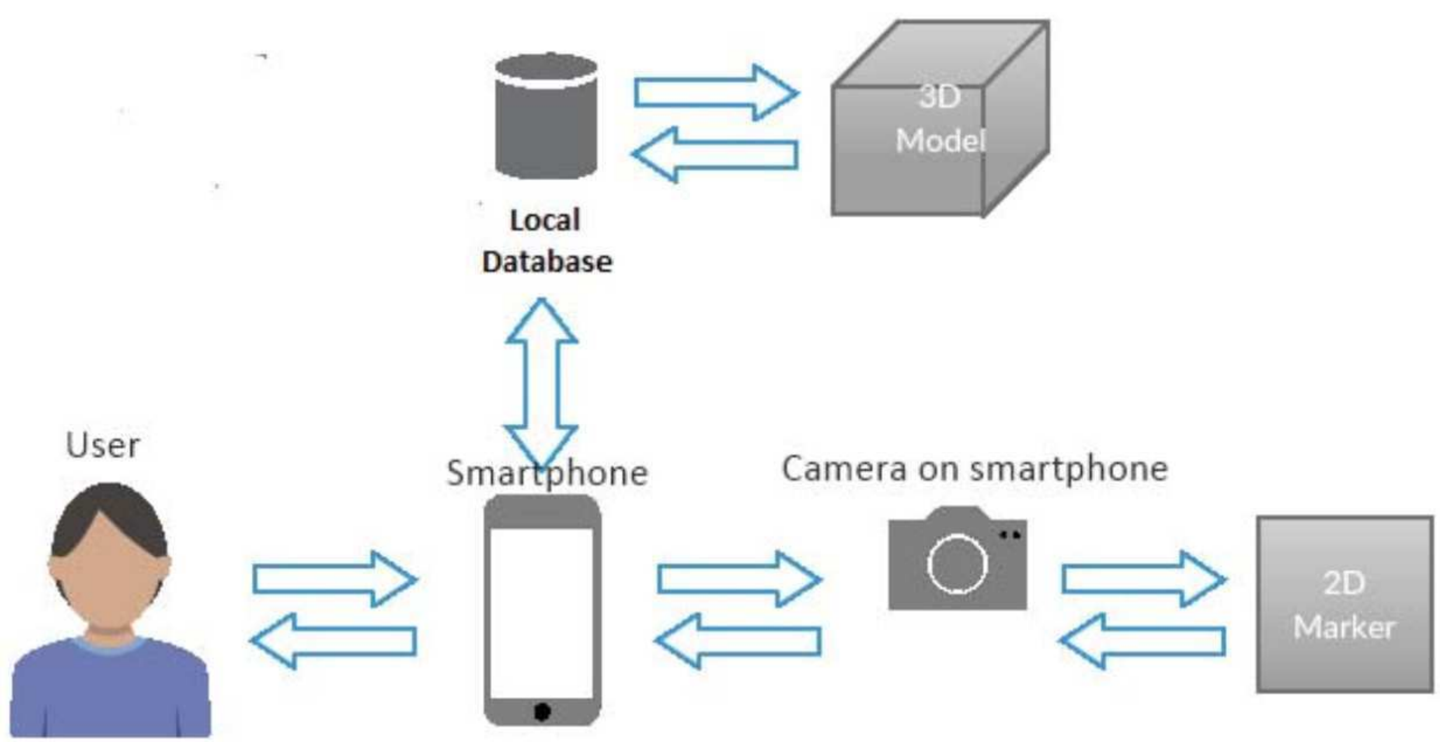


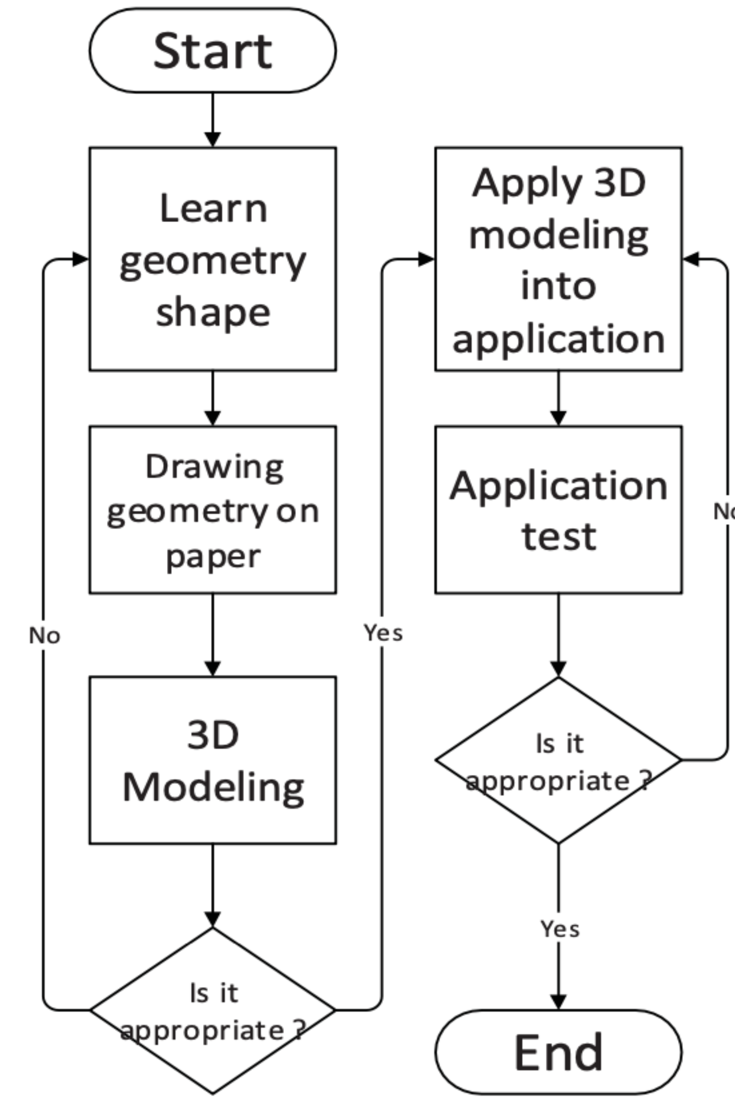


- AR (despite the title)
- Possibility of face-to-face collaboration
- Tests: high-school students

AR & mobile technologies







The 2nd East Indonesia Conference on Computer and Information Technology (EIConCIT) 2018

Augmented Reality Applied to Geometry Education

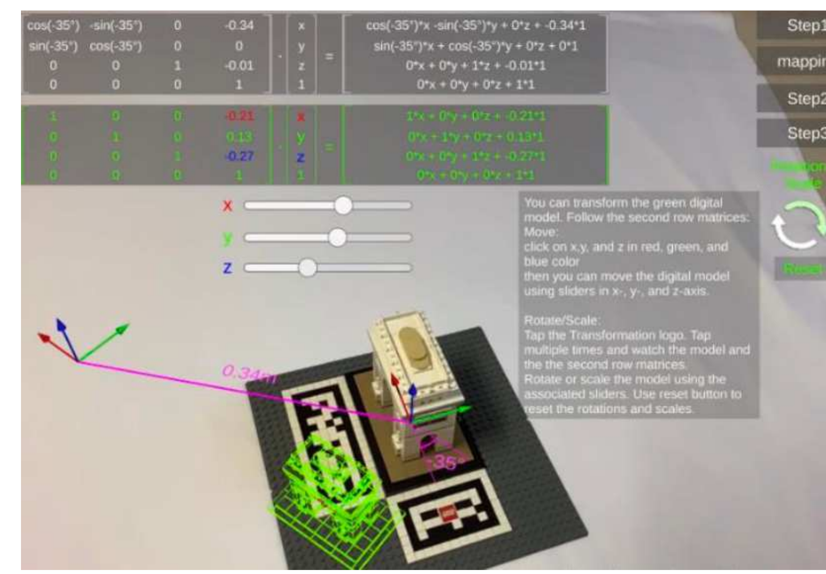
BRICKxAR_T (educational AR prototype)

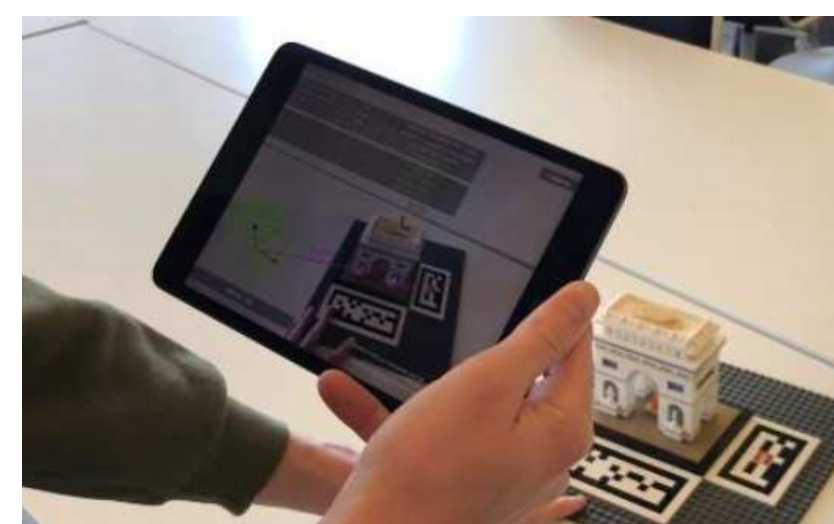
CAAD Futures 2021 Conference Proceedings, Springer.

Learning Geometric Transformations for Parametric Design: An Augmented Reality (AR)-Powered Approach

Zohreh Shaghaghian[1], Heather Burte[2], Dezhen Song[3], and Wei Yan[1]

- Purpose:
    - to help students understand the fundamental concepts of parametric modeling (digital design thinking, architecture)
    - to illustrate geometric transformation and the associated math functions so that students learn the mathematical logic behind the algorithmic thinking of parametric modeling
    - LEGO set is used within the AR intervention as a physical manipulative to support physical interaction and improve spatial skill through body gesture.
    - Effectiveness has not yet been tested



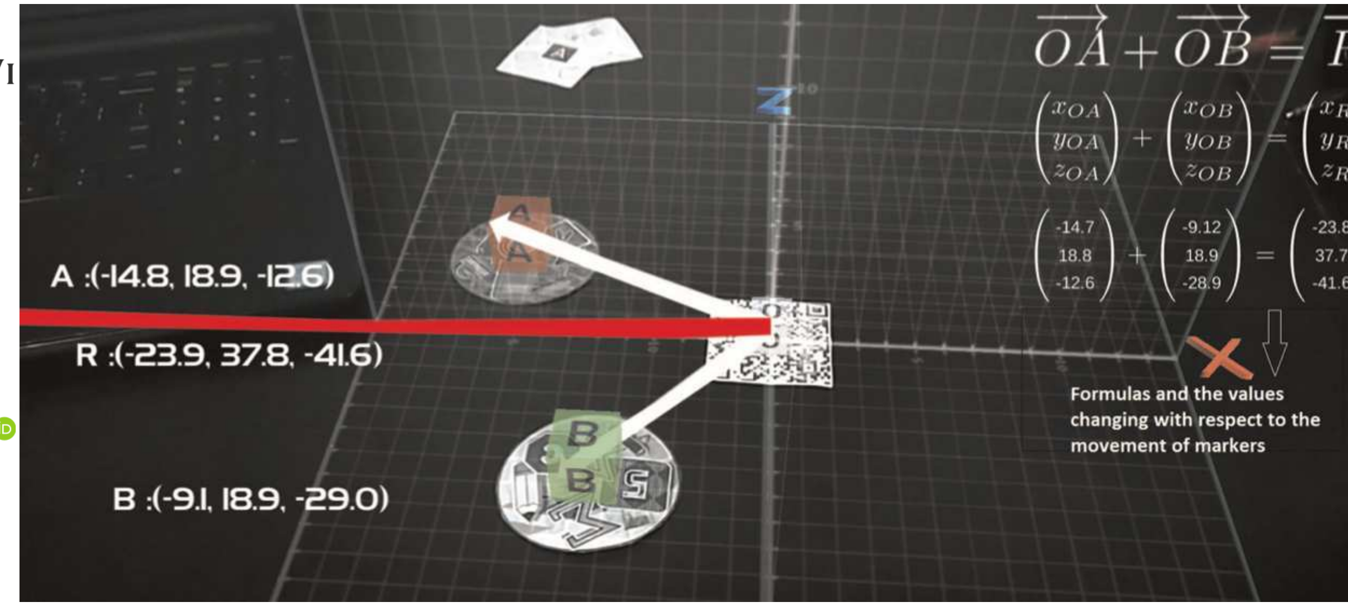


Received: 18 November 2020 | Revised: 1 April 2021 | Accepted: 15 April 2021
DOI: 10.1002/cae.22424

RESEARCH ARTICLE

Measuring effectiveness of augmented reality-based geometry learning assistant on memory retention abilities of the students in 3D geometry

Shubham Gargrish | Deepti P. Kaur | Archana Mantri | Gurjinder Singh
Bhanu Sharma



- Geometry Learning Assistant (GLA) was developed to teach geometry to students in a mathematics course.
- The app includes vector addition, cross product, position vector, direction ratios, and dot product with real-time examples for better learning.
- Tests: 1st-year polytechnic students
- Students of the experimental group were made to learn using AR-based GLA, whereas controls were treated using interactive simulation (IS).
- Learners from the experimental group have better memory retention abilities after 2 and 4 weeks of the learning activity.

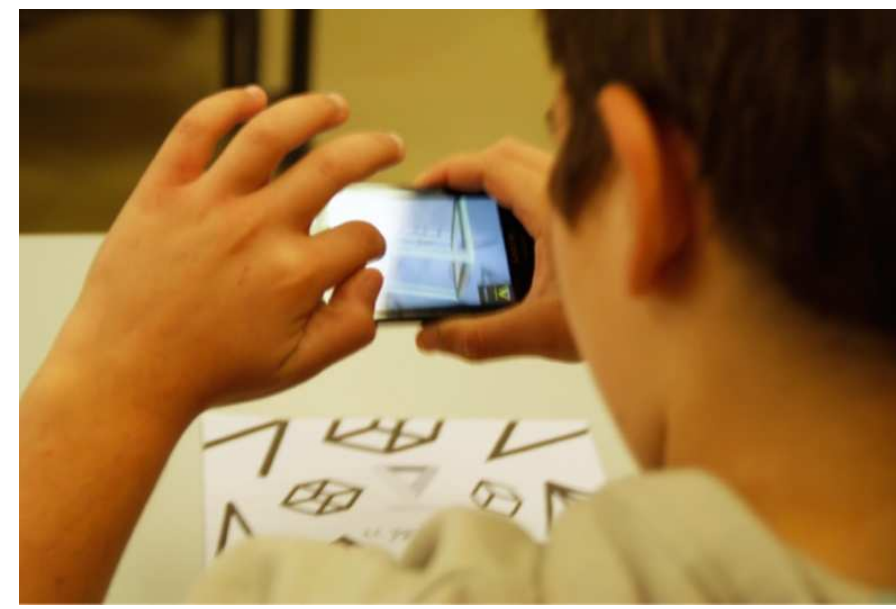
Mobile Learning:
Benefits of Augmented Reality in Geometry Teaching

Rui Leitão
Loughborough University, UK

João M.F. Rodrigues
Universidade do Algarve & LARSyS, Portugal

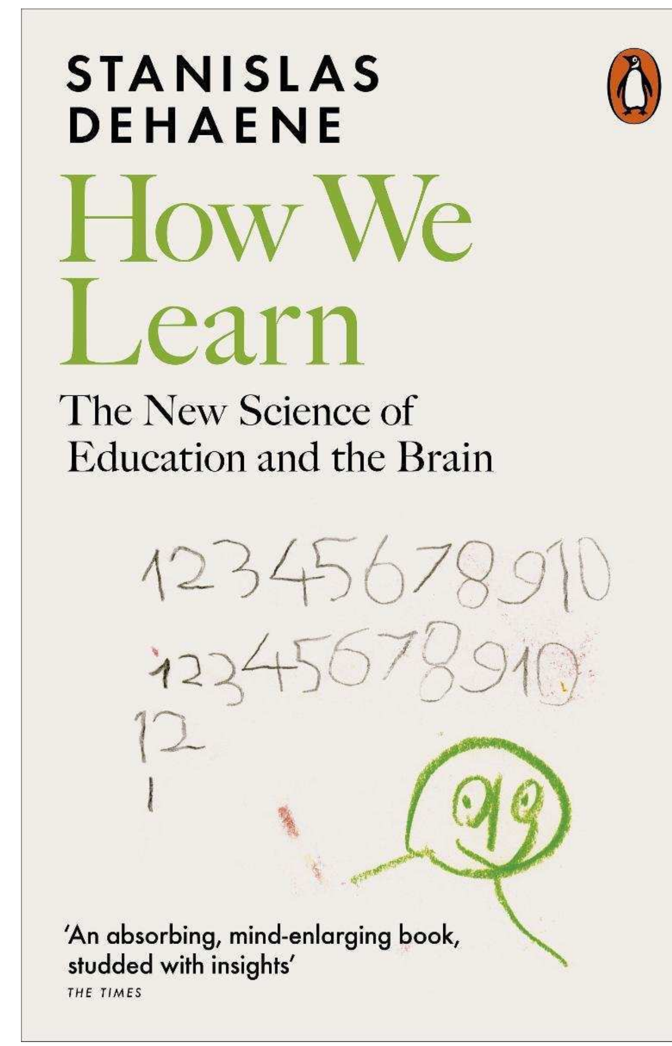
Adérito Fernandes Marques
Universidade Aberta, Portugal

- Designed for 8-10 years old children
- Aims:
  - a better visualization of geometric objects on a plane and in space
  - understanding of the properties of geometric solids
  - familiarization with the vocabulary of geometry
- The authors found that students have improved around 35% the hits of correct responses to the classification and differentiation between edge, vertex and face in 3D solids (in comparison to controls with verbal+whiteboard instructions)



Proposals of further research: Implementing trainings in younger children

- most of the current trainings are aimed at >10 years old children
- There are critical periods in cognitive development (language acquisition, mathematics learning)
- My hypothesis of critical period in geometry learning: 5-7 years old
- Particularly important in children with mathematics learning problems



Proposals of further research: Haptic trainings

- AR research/trainings (contrary to VR) include primarily purely-visual cues, and frequently neglect the possibility of implementing haptic interactions

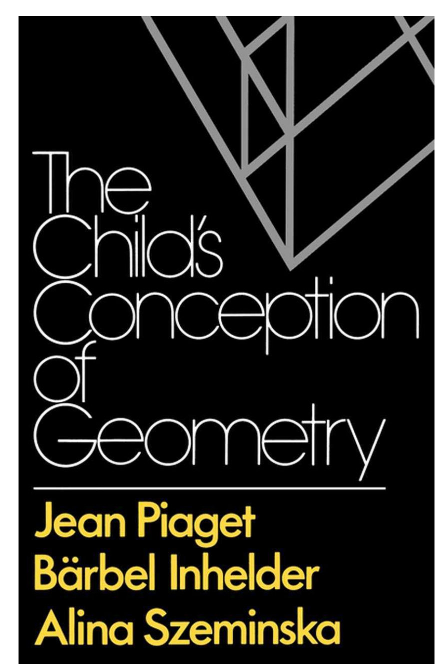
- Importance: geometry is not not purely visual!

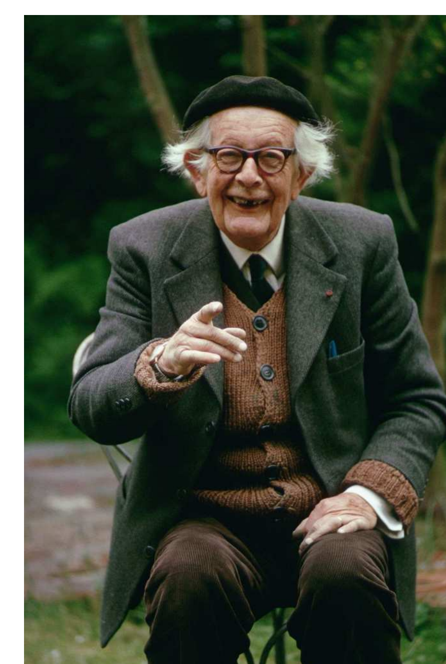
Does Spatial Navigation Have a Blind-Spot? Visiocentrism Is Not Enough to Explain the Navigational Behavior Comprehensively

*Mateusz Hohol[1,2], Bartosz Baran[3], Michał Krzyżowski[3] and Jacek Francikowski[3]**

frontiers in Behavioral Neuroscience

published: 18 August 2017
doi: 10.3389/fnbeh.2017.00154



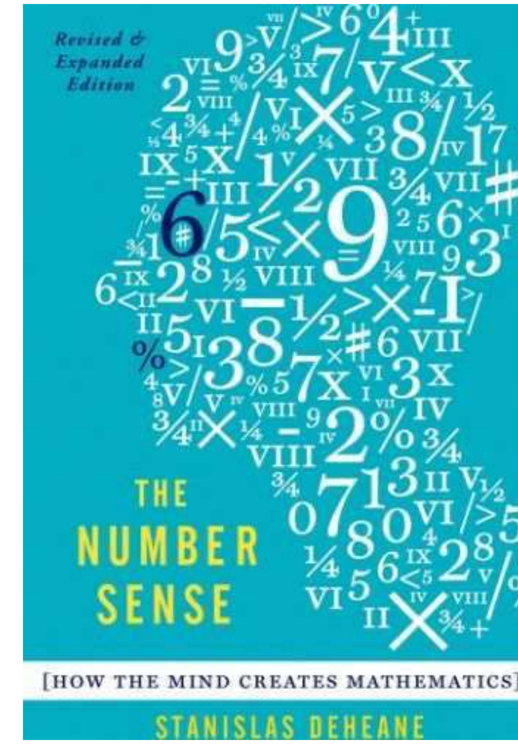


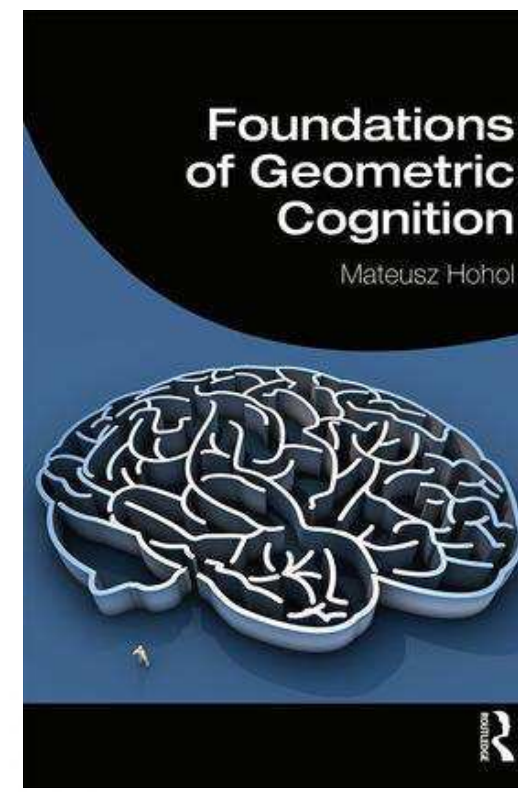
Proposals of further research: Combining geometric and numerical skills

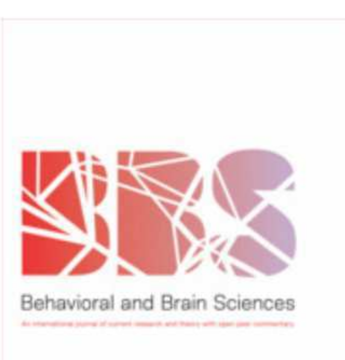
- Although core foundations of geometric and numerical cognition seem to be distinct (e.g., developmental dyscalculia does not affect geometric cognition) educational success depends of relevant combination of various domains of mathematics training

- Existing VR & AR educational tools engage geometric *or* numerical cognition

- Facilitating combination of geometry and numbers seems to be justified by more recent theoretical approaches







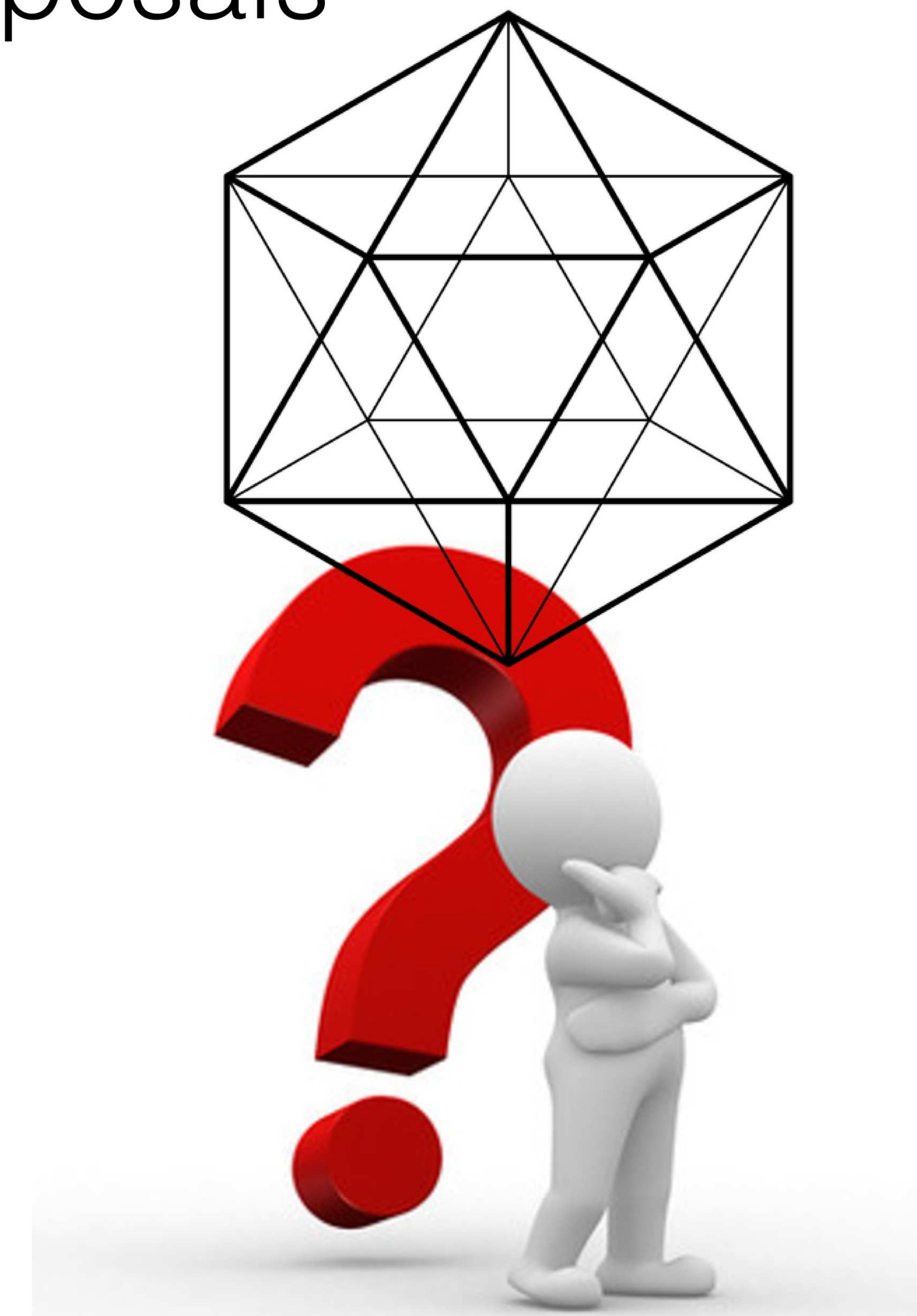
From "sense of number" to "sense of magnitude": The role of continuous magnitudes in numerical cognition

Published online by Cambridge University Press: **17 August 2016**

Tali Leibovich, Naama Katzin, Maayan Harel and Avishai Henik

Show author details

Other issues of existing trainings and new proposals



- Far transfer of learned geometric skills
- Sustainability of effects
- Lack of testing the depth of geometric concepts understanding in rigorous paradigms
- Not clear aims: digital education aid? facilitation of learning? real help for children with learning problems?
- Scenarios are relatively-fixed, and does not follow individual experiences and worries (possible usefulness of sensors like EGR in testing new interventions)
- Existing AR apps focus mainly on object geometry (neglecting layout geometry)

contact:
e-mail: mateusz@hohol.pl
web: www.hohol.pl/en

Thank you for your attention!

Please find my talk as a call to collaboration

JAGIELLONIAN UNIVERSITY IN KRAKÓW
Copernicus Center